PROPOSED TITLE OF THE RESEARCH

Certain investigations on some properties of analytic functions

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SYNOPSIS

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1 Introduction and definitions

For an entire function f, let $F(r) = max\{|f(z)| : |z| = r\}$. If f is nonconstant then $F(r)$ is strictly increasing and a continuous function of r and its inverse

$$F^{-1} : (|f(0)|, \infty) \to (0, \infty)$$

exists and $\lim\limits_{R\to\infty} F^{-1}(R) = \infty$. Let $f(s)$ be an entire function of the complex variable $s = \sigma + it$ defined by everywhere absolutely convergent Dirichlet series

$$\sum_{n=1}^{\infty} a_n e^{s\lambda_n} \tag{1.1}$$

where $0 < \lambda_n < \lambda_{n+1}(n \geq 1),\ \lambda_n \to \infty$ as $n \to \infty$ and $a_n's$ are complex constants.

Now we introduce the following definitions.

Definition 1.1. The relative L-Ritt order $\rho_g^L(f)$ of $f(s)$ with respect to a meromorphic $g(s)$ is defined as

$$\rho_g^L(f) = \inf\{\mu > 0 : T_f(\sigma) < [T_g(\sigma)L(\sigma)]^{\mu} \text{ for all large } \sigma\}$$

where $T_g(\sigma)$ is the Nevanlinna Characteristic function of $g(s)$.

Definition 1.2. Let f be entire and g be a meromorphic function which is not transcendental. Let $\alpha = inf\mu$ such that
$\int_{\sigma_0}^{\infty} \frac{T_f(\sigma)L(\sigma)}{[T_g(\sigma)L(\sigma)]^{\mu+1}}\, d\sigma,\ \sigma_0 \geq \sigma_0^{'} > 0$
converges.

Definition 1.3. Let f and g be two non-constant entire functions of n-complex variables and D be a bounded complete n-circular domain with centre at the origin in $\mathbb{C}^n$.
If p, q are positive integers such that $p > q \geq 1$ then the $(p,q)^{th}$ Gol'dberg order and $(p,q)^{th}$ Gol'dberg lower order are respectively denoted by $\rho_{f,D}^{[p,q]}$ and $\lambda_{f,D}^{[p,q]}$ and are defined by

$$\rho_{f,D}^{[p,q]} = \limsup_{R\to\infty} \frac{log^{[p]} M_{f,D}(R)}{log^{[q]} R}$$

and

$$\lambda_{f,D}^{[p,q]} = \liminf_{R\to\infty} \frac{log^{[p]} M_{f,D}(R)}{log^{[q]} R},\ p = 2, 3, 4, \ldots .$$

Definition 1.4. Let f and g be entire functions of n-complex variables and D be a bounded complete n-circular domain with centre at the origin in $\mathbb{C}^n$. Then $(p,q)^{th}$ relative Gol'dberg order $\rho_{g,D}^{[p,q]}(f)$ of f with respect to g in the domain D is defined by

$$\rho_{g,D}^{[p,q]}(f) = \limsup_{R\to\infty} \frac{log^{[p-1]} M_{g,D}^{-1}(M_{f,D}(R))}{log^{[q]} R}.$$

Similarly $(p,q)^{th}$ relative Gol'dberg lower order $\lambda_{g,D}^{[p,q]}(f)$ with respect to g in the domain D is defined by

$$\lambda_{g,D}^{[p,q]}(f) = \liminf_{R\to\infty} \frac{log^{[p-1]} M_{g,D}^{-1}(M_{f,D}(R))}{log^{[q]} R}.$$

We say that f is of $(p,q)^{th}$ regular growth if $\rho_{g,D}^{[p,q]}(f) = \lambda_{g,D}^{[p,q]}(f)$.

Definition 1.5. An entire function g is said to have property (A) if for any $\alpha > 1$ and for all large R,

$$\{M_{g,D}exp^{[p-1]}R\}^2 < \{M_{g,D}exp^{[p-1]}R^{\alpha}\}.$$

Definition 1.6. A nonconstant meromorphic function $g(s)$ is said to have the property(B) if for any $n > 1$ and large σ, $T_g(n\sigma) = O(T_g(\sigma))$.

2 Theorems

Using this definitions we have the following theorems such as sum theorem, product theorem, theorem on derivatives etc., which we state only.

Theorem 2.1. *Let f_1 and f_2 be entire functions having respective relative L-Ritt orders ${\rho_g}^L(f_1)$ and ${\rho_g}^L(f_2)$ with respect to meromorphic g.*

Then (i) ${\rho_g}^L(f_1 \pm f_2) \leq max\{{\rho_g}^L(f_1), {\rho_g}^L(f_2)\}$
and (ii) ${\rho_g}^L(f_1 f_2) \leq max\{{\rho_g}^L(f_1), {\rho_g}^L(f_2)\}$.

Theorem 2.2. *Let f be an entire function defined by (1.1) and g is transcendental such that g and g' satisfy property (B).*
Then $\rho_{g'}^L(f) = \rho_g^L(f)$.

Theorem 2.3. *If $\rho_g^L(f)$ be the relative L-Ritt order of f with respect to g and α is defined by definition (1.2), then $\rho_g^L(f)$ is finite if α is finite.*

Theorem 2.4. *Let f and g be entire functions of n-complex variables then $(p,q)^{th}$ relative Gol'dberg order of f with respect to g is independent of the choice of the domain D.*

Theorem 2.5. *Let f and g be entire functions of n complex variables such that*
$0 < \lambda_f^{[p+1,p]} \leq \rho_f^{[p+1,p]}$ *and* $0 < \lambda_g^{[p+1,p]} \leq \rho_g^{[p+1,p]}$.
Then $\frac{\lambda_f^{[p+1,p]}}{\rho_g^{[p+1,p]}} \leq \lambda_g^{[p+1,p]}(f) \leq min\{\frac{\lambda_f^{[p+1,p]}}{\lambda_g^{[p+1,p]}}, \frac{\rho_f^{[p+1,p]}}{\rho_g^{[p+1,p]}}\} \leq max\{\frac{\lambda_f^{[p+1,p]}}{\lambda_g^{[p+1,p]}}, \frac{\rho_f^{[p+1,p]}}{\rho_g^{[p+1,p]}}\} \leq \rho_g^{[p+1,p]}(f) \leq \frac{\rho_f^{[p+1,p]}}{\lambda_g^{[p+1,p]}}$.

Theorem 2.6. *Let f and g be entire functions of n complex variables such that $\rho_f^{[p+1,p]} = 0$ and $0 < \rho_g^{[p+1,p]} < \infty$. Then $\lambda_g^{[p+1,p]}(f) = 0$.*

Theorem 2.7. *Let f and g be entire functions of n complex variables such that $0 < \rho_f^{[p+1,p]} < \infty$ and $\rho_g^{[p+1,p]} = 0$. Then $\rho_g^{[p+1,p]}(f) = \infty$.*

Theorem 2.8. *Let f_1 and f_2 be two entire functions of $(p+1,p)^{th}$ relative Gol'dberg order $\rho_g^{[p+1,p]}(f_1)$ and $\rho_g^{[p+1,p]}(f_2)$ respectively. If g has the property (A), then the $(p+1,p)^{th}$ relative Gol'dberg order of $f_1.f_2$ is $\rho_g^{[p+1,p]}(f_1.f_2) \leq max\{\rho_g^{[p+1,p]}(f_1), \rho_g^{[p+1,p]}(f_2\}$.*

Theorem 2.9. *Let f be an entire function of $(p+1,p)^{th}$ relative Gol'dberg order $\rho_g^{[p+1,p]}(f) \neq 0$ and $P(z)$ be a polynomial. If g has the property (A), then the $(p+1,p)^{th}$ relative Gol'dberg order $\rho_g^{[p+1,p]}(f.P)$ of $f(z).P(z)$ is same as $\rho_g^{[p+1,p]}(f)$.*

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